AIAA JOURNAL
Vol. 30, No. 9, September 1992

Selection of Noisy Measurement Locations for Error Reduction in Static Parameter Identification

Masoud Sanayei,* Oladipo Onipede,† and Suresh R. Babu‡
Tufts University, Medford, Massachusetts 02155

An incomplete set of noisy static force and displacement measurements is used for parameter identification of structures at the element level. Measurement location and the level of accuracy in the measured data can drastically affect the accuracy of the identified parameters. A heuristic method is presented to select a limited number of degrees of freedom (DOF) to perform a successful parameter identification and to reduce the impact of measurement errors on the identified parameters. This pretest simulation uses an error sensitivity analysis to determine the effect of measurement errors on the parameter estimates. The selected DOF can be used for nondestructive testing and health monitoring of structures. Two numerical examples, one for a truss and one for a frame, are presented to demonstrate that using the measurements at the selected subset of DOF can limit the error in the parameter estimates.

Nomenclature

{DDOF}	= measured displacement DOF (NMD × 1)
$[\diagdown e_f \diagdown]$	= percentage force error (NSF × NSF)
$[\diagdown \hat{e}_f \diagdown]$	= absolute force error (NSF × NSF)
$[\diagdown e_u \diagdown]$	= percentage displacement error (NMD × NMD)
$[\diagdown \hat{e}_u \diagdown]$	= absolute displacement error (NMD × NMD)
$[F]$	= applied forces matrix (N × NSF)
$[F_a]$	= force matrix corresponding to $[u_a]$ (NMD × NSF)
$[F_b]$	= force matrix corresponding to $[u_b]$ (NUD × NSF)
$[F_m]$	= measured force matrix with errors (NMD × NSF)
$[F_t]$	= true force matrix, no errors (NMD × NSF)
{FDOF}	= applied force DOF (NSF × 1)
$[K]$	= global stiffness matrix (N × N)
$[K_{aa}]$	= submatrix of $[K]$; see Eq. (2) (NMD × NMD)
$[K_{ab}]$	= submatrix of $[K]$; see Eq. (2) (NMD × NUD)
$[K_{ba}]$	= submatrix of $[K]$; see Eq. (2) (NUD × NMD)
$[K_{bb}]$	= submatrix of $[K]$; see Eq. (2) (NUD × NUD)
m_1	= measured displacement DOF (no forces applied)
m_2	= measured displacement and applied force DOF
m_3	= applied force DOF (no displacements measured)
m_4	= no applied force and no measured displacement DOF
N	= total number of kinematic DOF $m_1 + m_2 + m_3 + m_4$
NM	= number of measurements = NMD × NSF
NMD	= number of measured displacement DOF $= m_1 + m_2$
NSF	= number of sets of applied forces $= m_2 + m_3$
NUD	= number of unmeasured displacement DOF $= m_3 + m_4$
NUP	= number of unknown parameters
$\{p\}$	= vector of parameter estimates (1 × NUP)
$\{\bar{p}\}$	= vector of mean of the parameter estimates (1 × NUP)
$\{p_t\}$	= vector of true values of parameters (1 × NUP)
$[R_f]$	= random numbers for applied forces (NMD × NSF)
$[R_u]$	= random numbers for measured displacements (NMD × NSF)
$[S_e]$	= error sensitivity matrix (NSF + NMD × NUP)
$[U]$	= displacements matrix (N × NSF)
$[U_a]$	= measured displacements matrix (NMD × NSF)
$[U_b]$	= unmeasured displacements matrix (NUD × NSF)
$[U_m]$	= measured displacement matrix with errors (NMD × NSF)
$[U_t]$	= true displacement matrix, no errors (NMD × NSF)

Received July 8, 1991; revision received Dec. 31, 1991; accepted for publication Jan. 15, 1992.

*Associate Professor, Department of Civil Engineering. Member AIAA.

†Former Research Assistant, Department of Civil Engineering; currently, Ph.D. candidate at UCLA, Department of Civil Engineering.

‡Research Assistant, Department of Civil Engineering.

Introduction

NONDESTRUCTIVE testing (NDT) is an integral part of parameter identification for damage assessment and health monitoring of structures. During the past two decades several finite element based static and dynamic parameter identification algorithms have been developed. These methods use various input test data such as measurements of forces, strains, displacements, accelerations, or frequencies. These measurements are used to estimate stiffness parameters of various structures such as aircraft, space stations, transmission towers, bridges, off-shore drilling platforms, and nuclear power plants. Factors that are necessary to guarantee a successful parameter estimation are the number of measurements, the location of measurements, and the accuracy of the measurements of the NDT.

The literature on parameter identification is quite extensive. Most of the research uses vibration measurements in terms of either time history or modal test data. Baruch,[1] Berman,[2] Kabe,[3] Adelman and Haftka,[4] Beck,[5] Chen and Garba,[6] Stubbs et al.,[7] and Hajela and Soeiro[8] have compiled and summarized the work done in the area of dynamic parameter identification. Hajela and Soeiro[8] presented a state-of-the-art paper on damage detection and classified the identification techniques as the equation error approach, the output error approach, and the minimum deviation approach. These methods are compared for their effectiveness in minimizing the difference between the analytical model and measured data. Both static and vibration measurements are used in this paper.

Static test data can also be used for damage assessment of structures. If the primary goal of the identification is to find element stiffness degradations, this goal can be achieved by a static parameter identification that is inherently easier than the dynamic problem. Sanayei and Nelson[9] presented a method of element stiffness identification using either a complete or incomplete set of static force and displacement measurements. Sanayei and Onipede[10] enhanced the static parameter identification algorithm to allow application of static forces to one subset of DOF(FDOF) and to measure displacements at another subset of DOF (DDOF). The two subsets, FDOF and DDOF, may completely overlap, partially overlap, or not overlap at all. This flexibility will allow complete freedom in the selection of FDOF and DDOF. For large structures it is more practical to measure strains than displacements. In a recent paper, Robson et al.[11] minimized the error between experimental and analytical static strain measurements to successfully identify the properties of element groups of highway bridges. The authors of Refs. 10 and 11 did not study the effect of measurement errors on the parameter estimates.

A determining factor in the success of parameter identification methods is the behavior of the algorithm in the presence of measurement errors. Hjelmstad et al.[12] presented an extensive report on damage assessment techniques using both static and modal simulated test data. Member stiffnesses were identified from complete and incomplete sets of noisy measured displacements. Hjelmstad and co-workers studied the behavior of their algorithm in the presence of different types of simulated errors.

A key pretest decision for NDT is the selection of the optimum location and the optimum number of measurement points for a successful parameter estimation. Skelton and Delorenzo[13] performed an input/output error analysis for control of modes of dynamic systems to maintain a static shape. They used a heuristic method to select the location of actuators and sensors. Haftka and Adelman[14] presented a paper on selection of actuator locations for static shape control of structures. Three heuristic methods, including the method of Ref. 13, were compared for the near-optimal selection of the sensor locations. References 13 and 14 are concerned with the control of the dynamic response of structures to maintain a static shape. Kammer[15] addresses the problem of selecting the given number of sensor locations from an initial much larger set such that the target mode partitions remain linearly independent. For the methods proposed in Refs. 13–15, there is no proof that the selected set of DOF is optimal. However, these methods seek a near-optimal solution.

A heuristic method is presented here for the selection of subsets of noisy FDOF and DDOF to reduce the error in the parameter estimates. In addition, the required precision of the static measurements that can keep the identification error below an acceptable level is determined.

Parameter Identification

If the applied forces and measured displacements contain errors, the parameter estimates will also contain errors. The parameter identification method presented in Ref. 10 is used in this paper for the selection of noisy force and displacement DOF to reduce the error in the identified parameters.

Reference 10 is briefly summarized here for completeness. A method was presented to perform parameter identification at the element level from incomplete static force and displacement measurements. The identified parameters (i.e., equivalent area and moment of inertia) were used to determine whether there is any degradation or failure of the critical components of various structures. This method preserved the topology and geometry of the linear elastic finite element model.

There are two different subsets of DOF for measurement purposes: the applied forces (FDOF) and the measured displacements (DDOF). These two sets of DOF may or may not overlap. NSF sets of forces are applied at FDOF, one set at a time, and NSF sets of displacements are measured at DDOF. These sets of applied forces and measured displacements are concatenated horizontally into a force matrix $[F]$ and a displacement matrix $[U]$ as shown by

$$[F] = [K][U] \tag{1}$$

Each column of the force matrix may contain a single load or multiple loads.

In fact, not all displacements in $[U]$ need to be measured. Therefore, Eq. (1) is partitioned into $[U_a]$ and $[U_b]$, the measured and unmeasured displacements:

$$\begin{bmatrix} F_a \\ \hline F_b \end{bmatrix} = \left[\begin{array}{c|c} K_{aa} & K_{ab} \\ \hline K_{ba} & K_{bb} \end{array}\right] \begin{bmatrix} U_a \\ \hline U_b \end{bmatrix} \tag{2}$$

The matrices $[F_a]$, $[F_b]$, and $[U_a]$ are obtained from the test data. $[U_b]$ was condensed from Eq. (2) to form a reduced force-displacement relationship. Then, the condensed equilibrium equation error was defined as an error function which is a nonlinear function of the stiffness parameters $\{p\}$. If the stiffness parameters are correct and there are no measurement errors, then the error function will be zero; otherwise, it will be nonzero.

A first-order Taylor series expansion was used to linearize the error function. The linearized error function was minimized with respect to the unknown parameters for parameter estimation. It was shown in Ref. 10 that, if no errors are present in the measurements, this method converges to the exact values of the parameters.

Since sets of forces are applied and sets of displacements are measured, the number of independent measurements (NIM) may be different from the number of unknown parameters (NUP). The NIM for single load cases is the total number of measurements minus the number of redundant measurements due to symmetry in the normalized matrix of measured displacements $[U_a]$:

$$\text{NIM} = (m_1 + m_2)(m_2 + m_3) - \tfrac{1}{2}m_2(m_2 - 1) \tag{3}$$

If NUP > NIM, a unique solution for the unknown parameters does not exist. If NUP = NIM, a direct inversion is utilized to compute the unknown parameters. If NUP < NIM, the method of least squares can be used.

Parameter Identification with Measurement Errors

In an actual test or field experiment, no matter how accurate the measurements are, errors will exist in both measured applied forces and measured displacements. These errors affect the values of the identified parameters and may even affect the convergence of the algorithm. A simplifying assumption is made that the only source of error in the parameter identification is in the measurements of forces and displacements. For actual tests other sources of error, such as modeling error, should be considered.

Parameter identification is performed in the presence of simulated measurement errors, and the magnitude of the errors in the measurements (input errors) and those in the identified parameters (output errors) are compared to establish a relationship between the input and output errors. Prior to any testing, the input/output error relationship can be used to determine accuracy limits on the measurements in order to achieve an acceptable error level in the identified parameters. In addition, it is essential to measure a subset of DOF that is not highly error sensitive.

Measurement Devices

Different types of force application and displacement measurement devices are available. For this study it is assumed that load cells are used for force measurements, linear variable differential transformers (LVDTS) for displacement measurements, and electronic tilt transducers can be used for rotations.

Load cells can be used to measure the magnitude of applied forces. The accuracy of a load cell is also proportional to the maximum capacity of the cell. Since a constant force is normally applied at each DOF for testing, it is possible to determine in advance the relative error expected in measurements of applied forces. Both force and displacement measurements may contain additional small errors due to nonlinearity, zero shift, repeatability, and temperature effects of the load cells and LVDTs.

LVDTs are accurate devices for measuring displacements. They are selected for each measured DOF based on the largest displacement expected at that DOF. The accuracy given with an LVDT is proportional to the maximum stroke of the LVDT. This implies that measurements much smaller than the maximum stroke may have large relative errors, whereas measurements close to the maximum stroke will have the smallest relative errors. Since displacements of varying magnitudes are measured with each LVDT, the relative error in each measured displacement will differ significantly. This makes it difficult to predict errors in the measured displacements. Electrolytic tilt meters can measure rotations with a precision comparable with LVDTs and suffer from similar types of errors.

Measurement Errors

The errors associated with different measuring devices can have different error distributions. Typical error distributions that are used to simulate test data are uniform and normal probability density functions (PDFs). A uniform PDF represents a banded type of error with equal probability of occurrence throughout the predefined limits. A normal PDF represents an error behavior that is not banded but has a higher probability of occurrence closer to the actual values.

Measurement errors may be added to the simulated measurements either as proportional errors or absolute errors. Proportional errors generate the largest error at the maximum value of the measurements, but absolute errors are added to the simulated measurements regardless of their magnitudes. The proportional errors are added to forces and displacements as

$$
\begin{aligned}
[U_m] &= [U_t] \odot (1 + [\backslash e_u \backslash][R_u]) \\
[F_m] &= [F_t] \odot (1 + [R_f][\backslash e_f \backslash])
\end{aligned} \tag{4}
$$

where $\odot$ represents element-by-element matrix multiplication (scalar). The absolute errors are added to forces and displacements as

$$
\begin{aligned}
[U_m] &= [U_t] + [\backslash \hat{e}_u \backslash][R_u] \\
[F_m] &= [F_t] + [R_f][\backslash \hat{e}_f \backslash]
\end{aligned} \tag{5}
$$

Since $[U_t]$ is a fully populated matrix, $[R_u]$ is a fully populated matrix of random numbers. However, $[F_t]$ is a diagonal matrix, therefore $[R_f]$ is also a diagonal matrix.

Both types of errors and both types of error distributions are insufficient to model different types of measurement errors. However, they can be used to study the effect of different types of errors on the identified parameters. In Ref. 12, the authors studied the effect of different types and distributions of measurement errors. Because of the simulation of different error types (proportional or absolute) and error distributions (uniform or normal), some differences exist in the input/output error behavior. However, any combination of error types and error distributions can be used to study the error behavior of the parameter identification algorithm.

For the method presented here, zero-mean uniform random numbers are generated and scaled by the corresponding percentage measurement error and then added proportionally to the force and displacement measurements for each measuring device. Each column of the measured displacement matrix represents a set of measurements, and each row represents all of the measurements made at one DOF. Errors induced by LVDTs contaminate the measured displacement matrix rowwise, whereas errors induced by load cells contaminate the measured displacement matrix columnwise.

If measurement errors do not exist and the displacement matrix is normalized with respect to the applied forces, the displacement matrix will be symmetric at the overlapping FDOF and DDOF. This symmetry is due to the Maxwell-Betti reciprocity law. When errors are present in either measurements of applied forces or displacements, the normalized measured displacement matrix will no longer be symmetric at the overlapping DOF. This violates the reciprocity law. In order to have a measured displacement matrix conforming to the Maxwell-Betti reciprocity law, symmetry is imposed on the overlapping DOF of the normalized measured displacement matrix.

Error Estimates in the Identified Parameters

As long as there are errors in the measurements, the algorithm will not exactly converge to the true values of the parameters. A Monte Carlo experiment is performed to determine the effect of the measurement errors on the parameter estimates. This approach is often used when the input/output relationship is complex, implicit, nonlinear, and does not easily lend itself to linear or nonlinear statistical estimation theories. The procedure for the Monte Carlo experiment is as follows:

1) Specify the number of observations (NOBS).
2) Specify the percentage error for each load cell and each LVDT.
3) Generate zero-mean uniform random numbers between − 1 and 1 for all measurements.
4) Multiply the percentage error of each measuring device by a random number generated in step 3.
5) Add scaled errors proportionally to the appropriate elements of the applied force and measured displacement matrices.
6) Normalize the displacements with respect to the applied forces and impose symmetry on the normalized measured displacement matrix at the overlapping FDOF and DDOF.
7) Run the parameter identification program using the contaminated force and displacement matrices. Store the parameter estimates.
8) Repeat steps 5–7 for each Monte Carlo observation.
9) Compute statistical properties of the parameter estimates from the Monte Carlo experiments.

The number of observations should be large enough to get a representative sample of the identified estimates. Each observation can be considered similar to an actual field test. The results of the Monte Carlo experiments at different input error levels can be used to study the input/output error behavior of the proposed parameter identification method.

Identification Error

Identification errors can potentially overshadow the actual values of the identified parameters. Based on the acceptable level of the error in the identified parameters, an acceptable input error level must be determined. This requires establishing an input/output error relationship. For the following truss example 100 Monte Carlo observations are performed to identify 10 unknown parameters. The uncertainty associated with this data must be quantified by a single value for the ease of comparing the input and output errors of different experiments (Ref. 12). The mean of parameter estimates is

$$
\{\bar{p}\} = \frac{1}{\mathrm{NOBS}} \sum_{i=1}^{\mathrm{NOBS}} \{p\}_i \tag{6}
$$

The bias of the parameter estimates in percentage of the true values is

$$\text{BIAS}(\{p\}) = \frac{\{\bar{p}\} - \{p_t\}}{\{p_t\}}(100) \tag{7}$$

The grand mean percentage error (GPE) is defined as the mean of the bias of the parameters in Eq. (7) as

$$\text{GPE} = \frac{1}{\text{NUP}} \sum_{j=1}^{\text{NUP}} |\text{BIAS}(p_j)| \tag{8}$$

The grand standard deviation percentage error (GSD) is defined as the standard deviation of the bias of parameters in Eq. (7) as

$$\text{GSD} = \sqrt{\frac{1}{\text{NUP}} \sum_{j=1}^{\text{NUP}} (|\text{BIAS}(p_j)| - \text{GPE})^2} \tag{9}$$

In this way the statistics of the error in the identified parameters are presented using one mean and one standard deviation that easily lend themselves to plotting. GPE and GSD can be potentially misleading because they do not show the maximum or minimum percentage error for each identified parameter. However, they show the overall identification percentage error for all parameter estimates of all Monte Carlo observations. If the percentage input error is the same for all the measurements, then the input/output error behavior can easily be plotted.

One of the most important aspects of NDT is the selection of force application locations (FDOF) and displacement measurement locations (DDOF). The number and the location of FDOF and DDOF can have an immense impact on the error in the parameter estimates. For a given percentage error in FDOF and DDOF, the error in the parameter estimates can potentially vary from small to medium to large and to extremely large identification errors, thus totally overshadowing the identified parameters.

The following example demonstrates the importance of performing an error sensitivity analysis prior to any NDT. Error sensitivity analysis will be used to find the most error tolerant DOF for measurements and to determine the level of acceptable error in the measurements.

Two-Story Truss Model

A two-story truss is illustrated in Fig. 1. Since truss models consist of axial members only, each element has one parameter, i.e., area. The physical geometries and the material properties are the following:

Modulus of elasticity of all elements: 30,000 ksi (205 GPa)

Initial value of all parameters (area): 5.0 in.^{2} (32.26 cm^2)

True value of all parameters (area): 3.0 in.^{2} (19.35 cm^2)

Four cases are simulated to compare the input/output error behavior of the two-story truss. Each case uses 4 FDOF and 4 DDOF for measurements. For each case a Monte Carlo experiment is performed using 100 observations. The input error at all FDOF and DDOF has a uniform distribution in the range

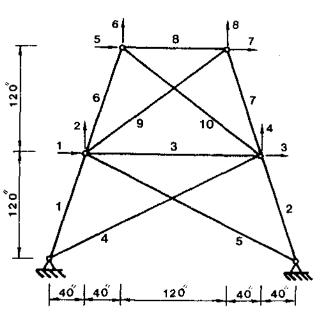

Fig. 1 Two-story truss.

Table 1 Truss identification error percentage for extreme cases (measurement error = 1%)

Case	Measured FDOF	Measured DDOF	GPE	GSD
A	1, 3, 5, 7	1, 3, 5, 7	11.18	18.23
B	5, 6, 7, 8	2, 4, 5, 7	0.36	0.33
C	1, 2, 3, 4	2, 4, 5, 7	43.64	50.20
D	1, 2, 3, 4	1, 3, 6, 8	4631.04	7995.05

of $\pm$ 1% with a zero mean and is proportionally added to the measurements. The grand mean percentage error of parameters (GPE) and the grand standard derivation error of parameters (GSD) are tabulated in Table 1.

All observations converged in two iterations for cases A, B, and C and all converged in nine iterations or less for case D. Case B has the smallest identification error, whereas cases A and C have large errors, but case D contains an extremely large error that is totally unacceptable. GPE for the best case is 0.36 and for the worst case is 4631.04, which clearly illustrates the importance of selection of an error-tolerant subset of DOF for NDT. Using the best engineering judgment, it is potentially possible to select an error sensitive subset of DOF for measurement and induce large errors in the identified parameters. Therefore, it is essential to perform an error sensitivity analysis for the selection of an error tolerant subset of DOF prior to any NDT and data collection.

Error Sensitivity Analysis: Best-in-Worst-Out Method

It is desirable to select the best FDOF and DDOF for measurement so that the errors in the parameter estimates are not very large compared to the measurement error. For any structural system with N degrees of freedom, the number of possible subsets (NS) of FDOF and DDOF for NDT is

$$\text{NS} = \left[\sum_{i=1}^{N} \binom{N}{i}\right]^2 \tag{10}$$

The two-story truss example presented has 8 DOF, which results in 65,025 possible combinations of FDOF and DDOF. However, parameter identification is only possible for the cases of Eq. (10) that NIM is greater than NUP. For practical reasons, FDOF and DDOF that are available for measurements are also limited, further reducing NS. NS will grow rapidly as the number of DOF in the finite element model increases. Since there is a large number of combinations of FDOF and DDOF, it is preferable to use a systematic method for selecting the optimum DOF for parameter identification. The optimal selection of the subset of DOF is an integer programming problem which is much more time consuming than a continuous optimization problem (Ref. 14).

A heuristic method for the selection of a set of FDOF and DDOF for parameter estimation with a low level of error in the identified parameters is proposed utilizing the algorithm in Ref. 10. This method utilizes error sensitivity analysis to determine the smallest subset of applied force and measured displacement DOF that will cause small errors in the identified parameters. This is accomplished by first finding the maximum errors in the identified parameters due to small changes in the measurements at each available DOF for testing. Then each of the FDOF and DDOF is dropped, one at a time, and the maximum errors in the identified parameters are determined again. The noisy DOF that when eliminated (not measured) cause small errors in the identified parameters can be eliminated with minimal impact on the accuracy of the parameter estimates. On the other hand, if dropping a DOF causes a large error in the identified parameters, this DOF should not be eliminated. This procedure is repeated until either the subset of measured DOF is so small that identification is no longer feasible, or, as the DOF are eliminated, the identification error grows to an unacceptable level.

Adding a small error to the measurements at one of the selected DOF and running the parameter identification program will give the relative change in the identified parameters due to the measurement error. This is the numerical equivalent of the partial derivative of each parameter with respect to the force and displacement measurements and is defined as the error sensitivity matrix, which is illustrated in Eq. (11). The largest element of each column of Eq. (11) gives an indication of the largest size of the error to be expected in the corresponding identified parameter:

$$[S_e] = \begin{bmatrix} \frac{\partial P_1}{\partial f_1} & \frac{\partial P_2}{\partial f_1} & \cdots & \frac{\partial P_i}{\partial f_1} \\ \frac{\partial P_1}{\partial f_2} & \frac{\partial P_2}{\partial f_2} & \cdots & \frac{\partial P_i}{\partial f_2} \\ \vdots & \vdots & & \vdots \\ \frac{\partial P_1}{\partial f_m} & \frac{\partial P_2}{\partial f_m} & \cdots & \frac{\partial P_i}{\partial f_m} \\ \hline \frac{\partial P_1}{\partial u_1} & \frac{\partial P_2}{\partial u_1} & \cdots & \frac{\partial P_i}{\partial u_1} \\ \frac{\partial P_1}{\partial u_2} & \frac{\partial P_2}{\partial u_2} & \cdots & \frac{\partial P_i}{\partial u_2} \\ \vdots & \vdots & & \vdots \\ \frac{\partial P_1}{\partial u_n} & \frac{\partial P_2}{\partial u_n} & \cdots & \frac{\partial P_i}{\partial u_n} \end{bmatrix} \tag{11}$$

In Eq. (11), i is the number of unknown parameters varying from 1 to NUP; m is the number of applied forces varying from 1 to NSF; and n is the number of measured displacements varying from 1 to NMD.

The procedure for the error sensitivity analysis is as follows:

1) Select all unknown parameters to be identified. The rest of the parameters are assumed to be known with a high level of confidence.

2) Select all *available* FDOFs and DDOFs for measurement.

3) Specify a low percentage error for all measuring devices. (The percentage error selected for the error sensitivity analysis should be small enough to ensure a linear input/output error behavior.)

4) For one of the selected DOF, add the predefined error of the designated measuring device to all measurements made with that device. All other measuring devices are assumed to be error free.

5) Run the parameter identification program using the measurements from step 4 and store the identified parameters.

6) Repeat steps 4 and 5 for each FDOF and DDOF selected. Form $[S_e]$ from Eq. (11).

7) Calculate and store the maximum percentage errors (MPE) for each of the identified parameters.

8) Drop one of the DOF selected in step 2 and repeat steps 4-7.

9) Repeat step 8 by restoring the dropped DOF and dropping a different DOF. This is repeated until all measured DOF have each been dropped once.

10) Summarize MPE similar to Table 2.

At the end of this procedure, the maximum percentage error in each parameter due to dropping a measured DOF can be compared with the initial run that had all the measured DOF present. If there are large changes in any of the parameters when a particular DOF is dropped, it means that some of the identified parameters are very sensitive to the elimination of that measured DOF. Dropping the most error-sensitive DOF will cause large errors in the parameters, whereas eliminating the least error-sensitive DOF will cause small errors in the identified parameters.

In summary, first an acceptable error bound for the identified parameters must be set. Then the measured DOF that when dropped create very small errors in the identified parameters are eliminated. Using the reduced subset of measured DOF, another error sensitivity analysis can be performed. Again, the measured DOF that when dropped create very small errors in the identified parameters are eliminated. This procedure should be repeated until all of the FDOF and DDOF that cause errors within the established error bound are eliminated. The remaining FDOF and DDOF will be the smallest subset of DOF that will cause errors within the acceptable error bound in the identified parameters. Parameter identification using a limited number of force and displacement measurements is a highly nonlinear process. Therefore, it is not recommended to drop all eligible DOF in one step. Instead, in each error sensitivity analysis only one DOF must be eliminated (unless a group of DOF has the same low error sensitivity). In this paper this method is called the "Best in/worst out" method.

At present, there is no proof that the selected subset of DOF is optimal. However, the heuristic method presented assists in the selection of a subset of measured noisy DOF that cause relatively small errors in the parameter estimates and greatly reduces the computational burden of the selection of the best DOF for NDT. Although the selected subset of DOF may not be the optimal set, it will be at least close enough to it (near-

Table 2 Results of truss error sensitivity analysis (case 1)

Eliminated		MPE for a 1% measurement error										
FDOF	DDOF	$A1$	$A2$	$A3$	$A4$	$A5$	$A6$	$A7$	$A8$	$A9$	$A10$	MPEA
0	0	1.0	1.0	0.6	−1.5	−1.5	−1.1	−1.1	0.6	−2.0	−2.0	−2.0
1	0	1.2	−1.1	2.4	−1.2	1.7	1.1	1.1	0.6	2.5	−2.1	2.5
2	0	0.8	1.0	−0.6	−1.5	−1.7	1.8	1.1	0.6	2.2	−2.0	2.2
3	0	−1.1	1.2	2.4	1.7	−1.2	1.1	1.1	0.6	−2.1	2.5	2.5
4	0	1.0	0.8	−0.6	−1.7	−1.5	1.1	1.8	0.6	−2.0	2.2	2.2
5	0	1.2	1.2	−0.7	1.6	−1.5	−1.2	1.1	7.5	−0.9	−1.1	7.5
6	0	1.4	1.0	0.7	1.5	−1.6	−3.4	−1.1	0.6	−2.2	2.8	−3.4
7	0	1.2	1.2	−0.7	−1.5	1.6	1.1	−1.2	7.5	−1.1	−0.9	7.5
8	0	1.0	1.4	0.7	−1.6	1.5	−1.1	−3.4	0.6	2.8	−2.2	−3.4
0	1	−1.0	−1.0	−1.6	1.5	1.0	−1.1	1.1	0.7	−3.6	2.3	−3.6
0	2	−0.5	1.0	0.6	−1.5	1.7	−1.8	−1.1	0.7	−2.4	−2.1	−2.4
0	3	−1.0	−1.0	−1.6	1.0	1.5	1.1	−1.1	0.7	2.3	−3.6	−3.6
0	4	1.0	−0.5	0.6	1.7	−1.5	−1.1	−1.8	0.7	−2.1	−2.4	−2.4
0	5	1.0	1.1	−0.7	1.5	1.6	1.2	1.1	−4.6	1.7	1.4	−4.6
0	6	0.9	1.0	0.7	−1.4	−1.5	3.3	−1.1	0.7	−2.2	−3.0	3.3
0	7	1.1	1.0	−0.7	1.6	1.5	1.1	1.2	−4.6	1.4	1.7	−4.6
0	8	1.0	0.9	0.7	−1.5	−1.4	−1.1	3.3	0.7	−3.0	−2.2	3.3

Measured DOF: FDOF = 1 2 3 4 5 6 7 8 and DDOF = 1 2 3 4 5 6 7 8.

optimal) to allow a successful parameter identification with low identification errors.

Error Sensitivity Analysis for the Two-Story Truss

Table 2 shows the results of an error sensitivity analysis for the truss example. All DOF are initially assumed to be available for measurements, and all 10 parameters (i.e., the equivalent areas) are assumed to be unknown. The first row of the data shows the maximum percentage error for each parameter when none of the DOFs are dropped. The subsequent rows show the maximum percentage error in the identified parameters when one of the DOF is eliminated from the measured DOF. The first two columns of the table show the DOF eliminated to examine the effect of the measurement errors on the identified parameters. The next 10 columns show the maximum percentage error (MPE) for each identified parameter. The last column is the maximum percentage error for all 10 parameters (MPEA). Since it is desirable to limit the maximum error in the parameter estimates, MPEA is ranked to drop the least error-sensitive DOF.

Table 2 is the result of one computer run representing the error sensitivities when different DOF are eliminated. The first row of this Table shows that, if all DOF are measured with a 1% input error, MPEA for the 10 identified parameters is −2.0%. The DOF that by their elimination from the set of measured DOF have the smallest impact on the identification error are FDOF 2 and 4 with MPEA of 2.2%. The rest of the DOF, if eliminated, will cause an error >2.2%.

Table 3 shows the results of a subsequent error sensitivity analysis after FDOF 2 and 4 are eliminated. The first row of the data shows that MPEA is −2.1% for the remaining measured DOF. Based on Table 3, the best DOF to drop from the measured subset of DOF are DDOF 2 and 4 with an MPE of 2.5% for each DOF.

Table 4 shows that eliminating FDOF 2 and 4 plus DDOF 2 and 4 will increase MPEA to −2.6% for a 1% measurement error. Next, dropping FDOF 1 and 3 will cause the least MPEA of 3.5%. It is clear that the worst DOF to eliminate are FDOF 6 and 8 because each elimination will cause a maximum of −11.4% error in one of the parameter estimates.

Table 5 lists the result of the error sensitivity analysis with FDOF 1 and 3 dropped along with all of the previously dropped DOF. MPEA for the remaining DOF is increased to −3.5%. The drop analysis shows that it is best to drop DDOF 5 and 7 with an MPEA of 3.3%. The worst DOF to eliminate are FDOF 6 and 8 and DDOF 1 and 3.

Table 6 shows the error sensitivity analysis results with the remaining measured FDOF of 5, 6, 7, and 8 and DDOF of 1, 3, 6, and 8. A 1% measurement error will cause an MPEA of −2.5% in the parameter estimates. The subsequent drop analysis and MPEA show that dropping any of the remaining DOF will cause a large error in the parameter estimates.

Table 6 shows that, if any of DOF are to be eliminated, they should be FDOF 5 and 7. Eliminating both DOF will cause NIM to reduce to 7, which is less than the number of unknown parameters, 10. In this case parameter identification is not possible. Therefore, dropping only FDOF 5 is recommended, thus reducing NIM to 11, which is still greater than NUP, making the identification possible. If FDOF 5 is eliminated (case 6), it is clear from Table 6 that MPEA will increase to 15.8%.

Table 3 Results of truss error sensitivity analysis (case 2)

Eliminated		MPE for a 1% measurement error										
FDOF	DDOF	$A1$	$A2$	$A3$	$A4$	$A5$	$A6$	$A7$	$A8$	$A9$	$A10$	MPEA
0	0	−0.8	−0.8	−0.6	1.7	1.7	−1.8	−1.8	−0.6	−2.1	−2.1	−2.1
1	0	1.0	−0.8	−2.2	1.5	−2.3	−1.8	−1.8	0.6	−2.8	−2.0	−2.8
3	0	−0.8	1.0	−2.2	−2.3	1.5	−1.8	−1.8	0.6	−2.0	−2.8	−2.8
5	0	−0.9	−1.0	0.7	−1.8	1.7	−1.8	−1.9	−7.4	0.6	−0.9	−7.4
6	0	−1.1	0.9	−0.7	1.7	1.9	−3.5	−1.8	−0.6	−2.2	−3.2	−3.5
7	0	−1.0	−0.9	0.7	1.7	−1.8	−1.9	−1.8	−7.4	−0.9	0.6	−7.4
8	0	0.9	−1.1	−0.7	1.9	1.7	−1.8	−3.5	−0.6	−3.2	−2.2	−3.5
0	1	−1.1	−0.9	1.4	−1.7	0.7	−1.7	−1.8	−0.7	3.5	−2.5	3.5
0	2	0.6	−0.8	−0.6	1.7	1.9	1.8	−1.8	−0.6	2.5	−2.3	2.5
0	3	−0.9	−1.1	1.4	0.7	−1.7	−1.8	−1.7	−0.7	−2.5	3.5	3.5
0	4	−0.8	0.6	−0.6	1.9	1.7	−1.8	1.8	−0.6	−2.3	2.5	2.5
0	5	−0.8	−0.8	0.7	−1.8	−1.8	−1.9	−1.8	3.1	−1.8	−1.4	3.1
0	6	−0.8	0.9	−0.6	1.7	1.8	−3.7	−1.8	−0.6	−2.2	−3.2	−3.7
0	7	−0.8	−0.8	0.7	−1.8	−1.8	−1.8	−1.9	3.1	−1.4	−1.8	3.1
0	8	0.9	−0.8	−0.6	1.8	1.7	−1.8	−3.7	−0.6	−3.2	−2.2	−3.7

Measured DOF: FDOF = 1 3 5 6 7 8 and DDOF = 1 2 3 4 5 6 7 8.

Table 4 Results of truss error sensitivity analysis (case 3)

Eliminated		MPE for a 1% measurement error										
FDOF	DDOF	$A1$	$A2$	$A3$	$A4$	$A5$	$A6$	$A7$	$A8$	$A9$	$A10$	MPEA
0	0	−0.6	−0.6	−0.6	−1.9	−1.9	−1.8	−1.8	0.6	−2.6	−2.6	−2.6
1	0	−0.6	0.7	−2.4	1.8	3.1	−1.8	−1.8	−0.7	3.5	−2.9	3.5
3	0	0.7	−0.6	−2.4	3.1	1.8	−1.8	−1.8	−0.7	−2.9	3.5	3.5
5	0	−0.9	1.3	−0.7	2.0	−1.9	−2.2	2.2	7.3	−0.6	−1.1	7.3
6	0	−4.2	0.6	0.6	1.8	−1.9	−11.4	2.0	0.9	1.4	3.7	−11.4
7	0	1.3	−0.9	−0.7	−1.9	2.0	2.2	−2.2	7.3	−1.1	−0.6	7.3
8	0	0.6	−4.2	0.6	−1.9	1.8	2.0	−11.4	0.9	3.7	1.4	−11.4
0	1	1.0	−0.6	−1.9	−2.2	4.3	−1.8	1.8	0.7	−3.9	−2.9	4.3
0	3	−0.6	1.0	−1.9	4.3	−2.2	1.8	−1.8	0.7	−2.9	−3.9	4.3
0	5	1.5	1.3	−0.6	1.9	2.0	2.3	−2.6	−4.8	1.0	1.5	−4.8
0	6	2.9	−0.8	0.6	−1.8	−1.9	−6.4	−1.8	0.9	−1.7	−4.4	−6.4
0	7	1.3	1.5	−0.6	2.0	1.9	−2.6	2.3	−4.8	1.5	1.0	−4.8
0	8	−0.8	2.9	0.6	−1.9	−1.8	−1.8	−6.4	0.9	−4.4	−1.7	−6.4

Measured DOF: FDOF = 1 3 5 6 7 8 and DDOF = 1 3 5 6 7 8.

Table 5 Results of truss error sensitivity analysis (case 4)

Eliminated FDOF	Eliminated DDOF	MPE for a 1% measurement error A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	MPEA
0	0	−0.7	−0.7	−0.9	−3.0	−3.0	1.8	1.8	−0.6	−3.5	−3.5	−3.5
5	0	1.0	−1.6	−1.1	−2.8	−2.5	2.1	−2.2	8.7	−1.6	−1.7	8.7
6	0	7.4	−0.9	−2.1	3.9	−3.8	17.7	−1.9	1.3	−1.2	−5.1	17.7
7	0	−1.6	1.0	−1.1	−2.5	−2.8	−2.2	2.1	8.7	−1.7	−1.6	8.7
8	0	−0.9	7.4	−2.1	−3.8	3.9	−1.9	17.7	1.3	−5.1	−1.2	17.7
0	1	3.2	1.1	16.3	−10.0	19.8	−3.2	−1.7	1.0	−2.5	−3.7	19.8
0	3	1.1	3.2	16.3	19.8	−10.0	−1.7	−3.2	1.0	−3.7	−2.5	19.8
0	5	−1.8	−1.3	−1.0	−2.4	−2.5	−2.2	2.5	3.3	1.3	−2.0	3.3
0	6	−3.3	0.8	−1.2	3.6	−3.4	7.2	1.9	−1.0	2.0	5.2	7.2
0	7	−1.3	−1.8	−1.0	−2.5	−2.4	2.5	−2.2	3.3	−2.0	1.3	3.3
0	8	0.8	−3.3	−1.2	−3.4	3.6	1.9	7.2	−1.0	5.2	2.0	7.2

Measured DOF: FDOF = 5 6 7 8 and DDOF = 1 3 5 6 7 8.

Table 6 Results of truss error sensitivity analysis (case 5)

Eliminated FDOF	Eliminated DDOF	MPE for a 1% measurement error A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	MPEA
0	0	−1.0	−1.0	−1.0	−2.5	−2.5	1.8	1.8	−1.0	1.7	1.7	−2.5
5	0	0.8	0.8	−1.2	1.0	1.3	2.0	2.0	15.8	1.5	1.4	15.8
6	0	4.0	−3.2	−5.3	−8.1	9.2	27.2	3.6	−6.6	−2.0	−21.0	27.2
7	0	0.8	0.8	−1.2	1.3	1.0	2.0	2.0	15.8	1.4	1.5	15.8
8	0	−3.2	4.0	−5.3	9.2	−8.1	3.6	27.2	−6.6	−21.0	−2.0	27.2
0	1	3.3	2.1	−6.9	−6.5	−15.2	−3.1	2.4	16.9	3.3	−2.6	16.9
0	3	2.1	3.3	−6.9	−15.2	−6.5	2.4	−3.1	16.9	−2.6	3.3	16.9
0	6	4.1	4.1	5.4	8.6	9.3	−8.1	4.7	−8.1	−2.7	−25.9	−25.9
0	8	4.1	4.1	5.4	9.3	8.6	4.7	−8.1	−8.1	−25.9	−2.7	−25.9

Measured DOF: FDOF = 5 6 7 8 and DDOF = 1 3 6 8.

Table 7 Truss identification error percentages for all cases (measurement error = 1%)

Case	Measured FDOF	Measured DDOF	NIM	GPE	GSD
1	1, 2, 3, 4, 5, 6, 7, 8	1, 2, 3, 4, 5, 6, 7, 8	36	0.121	0.098
2	1, 3, 5, 6, 7, 8	1, 2, 3, 4, 5, 6, 7, 8	33	0.240	0.130
3	1, 3, 5, 6, 7, 8	1, 3, 5, 6, 7, 8	21	0.139	0.119
4	5, 6, 7, 8	1, 3, 5, 6, 7, 8	18	0.143	0.096
5	5, 6, 7, 8	1, 3, 6, 8	15	0.206	0.249
6	6, 7, 8	1, 3, 6, 8	11	0.209	0.177

From the first rows of Tables 2–6 it is observed that the identification error slightly increased as DOF were eliminated. Identification error did not increase drastically because a careful error sensitivity analysis was performed to determine the least error sensitive DOF to eliminate. If an error sensitivity analysis is not performed, it is possible to end up with a set of measured DOF similar to case D of Table 1 with extremely large errors in the identified parameters.

Cases 1–6 give acceptable sets of DOF for measurement that allow identification with reasonably low errors in the parameter estimates. Now there is a tradeoff between the number of measured DOF and the accuracy of the parameter estimates. In order to select the best among these six cases and also determine the level of acceptable measurement error, a Monte Carlo experiment is performed for each case to study the input/output error behavior.

Error Behavior of the Two-Story Truss

In the preceding example errors in the identified parameters are induced by adding a 1% error to only one measured DOF. This was done to see how measurements at each FDOF or DDOF contribute to the errors in the identified parameters. However, in an actual test, random errors are present in all measurements, and these errors will not all be at their maximum values. In order to examine the effect of measurement errors on the identified parameter, a Monte Carlo experiment is performed for each of the six cases presented.

The two-story truss illustrated in Fig. 1 is used to investigate the impact of force and displacement measurement errors on the identified parameters. All the geometrical and physical properties of the structure are taken to be the same as before. The measurements of forces and displacements have a uniform-proportional random error in the range of 1%. This bound provides a reasonable and practical level of precision. One hundred Monte Carlo observations are performed, and the results are presented in Table 7.

All observations for these six experiments converged in two iterations. In Table 7 the case number, measured FDOF, measured DDOF, number of independent measurements, the grand percentage error, and the grand percentage standard deviation are given. The percentage errors in the identified parameters are low and reasonable. This will allow comparisons between the parameter estimates and their initial values for damage assessment. Table 7 shows that cases 2, 5, and 6 have relatively larger errors, whereas cases 1, 3, and 4 have smaller errors in the parameter estimates. Monte Carlo experiments are performed for these cases at different input error levels to determine the acceptable input error and to select one of the cases presented in Table 7 for potential NDT.

Input-Output Error Behavior

In order to ensure a successful parameter identification, it is important that the specified level of accuracy in all designated measuring devices be maintained. By examining the input/

output error behavior, it is possible to select the level of acceptable input error. For the truss example this relationship was determined by performing a series of Monte Carlo experiments at input error levels of 0.1, 0.25, 0.5, 0.75, 1.0, 2.5, 5.0, and 7.5%. GPE and GSD for 100 Monte Carlo observations for all six cases are given in Tables 8 and 9 and illustrated in Figs. 2 and 3, respectively. It is observed that the input/output error relationship varied from linear to slightly nonlinear, to a highly nonlinear, and then to an undeterminable relationship due to divergence of the program.

All cases performed reasonably well below a 1% measurement error. Cases 1–4 performed well up to a 7.5% input error. The selection of a set of DOF should be based on the acceptable errors in the identified parameters and the precision of the available measuring devices. Case 1, representing a complete set of measurements, and case 4, representing an incomplete set of measurements, performed similarly well. Since a complete set of measurements is not desirable, case 4 is recommended for potential testing. The acceptable measurement error is selected at 2.5% with GPE and GSD equal to 0.457 and 0.313%, respectively.

From these results it is concluded that the error sensitivity analysis is an essential process for selecting a subset of DOF for measurements to perform a successful parameter identification. The error sensitivity algorithm presented in this paper selected significantly better subsets of DOF (cases 1–6) compared to cases A–D of Table 1 for parameter estimation.

Two-Story Frame Model

A two-story frame is illustrated in Fig. 4. The physical geometries and the material properties are the following:

Modulus of elasticity of all elements: 205 Gpa

Initial value of all parameters (moment of inertia): 1500 mm^4

True value of all parameters (moment of inertia): 1000 mm^4

This small scale frame is prepared for future testing and damage assessment at Tufts University. In this example a pretest study is performed to select a subset of DOF and an acceptable measurement error level to ensure a successful parameter identification.

Cross-sectional properties used to define two-dimensional frame elements are areas and moment of inertias. Since frame elements are axially much stiffer compared to bending, all frame components are modeled as axially rigid elements. Therefore, each frame element has one parameter to be identified which is the moment of inertia. Furthermore, inclusion of axial DOF will create a stiffness matrix that contains high axial and relatively low bending stiffnesses values. This situation can cause an ill-conditioned sensitivity matrix that can lead to very large identification errors.[10] Therefore, this frame is modeled using eight frame elements.

The two-story frame example presented here has 10 DOFs, which results in 1,046,529 possible subsets of FODF and DDOF [Eq. (10)]. It is evident that an error sensitivity analysis is needed for the selection of an error-tolerant subset of FDOF and DDOF for NDT.

An error sensitivity analysis using the proposed best in/worst out method is performed. For brevity, only MPEA (similar to the last columns of Tables 2–6 for the truss exam-

Table 8 Truss grand mean percentage error, GPE

I_e	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
0.10	0.012	0.024	0.014	0.012	0.014	0.022
0.25	0.030	0.059	0.035	0.031	0.037	0.054
0.50	0.060	0.119	0.069	0.065	0.084	0.106
0.75	0.091	0.179	0.104	0.102	0.141	0.157
1.00	0.121	0.240	0.139	0.143	0.206	0.209
2.50	0.310	0.622	0.350	0.457	1.131	1.830
5.00	0.652	1.324	0.733	1.367	5.623	13.872
7.50	1.192	2.151	1.221	2.788	21.095	40.836

Table 9 Truss grand standard deviation percentage error, GSD

I_e	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
0.10	0.010	0.013	0.013	0.010	0.008	0.023
0.25	0.025	0.032	0.032	0.025	0.027	0.054
0.50	0.049	0.064	0.062	0.049	0.077	0.099
0.75	0.074	0.097	0.091	0.072	0.152	0.139
1.00	0.098	0.130	0.119	0.096	0.249	0.177
2.50	0.250	0.325	0.272	0.313	2.380	4.360
5.00	0.589	0.722	0.514	1.041	14.491	38.672
7.50	1.369	1.352	0.827	2.528	59.196	117.480

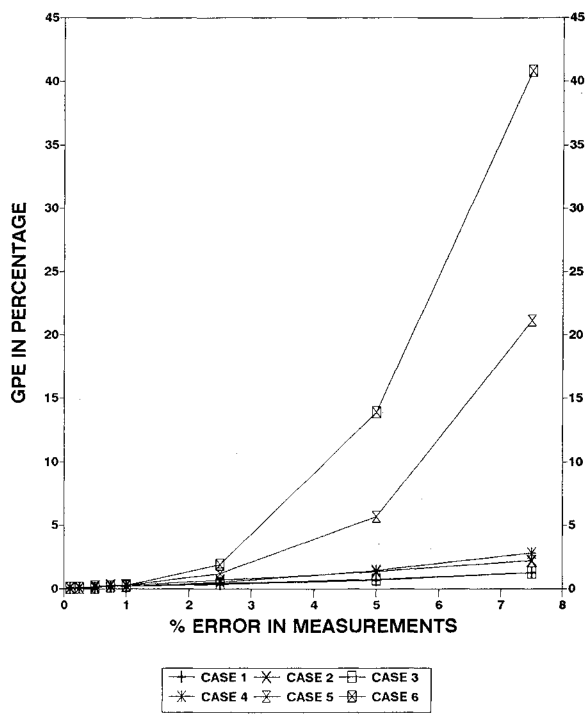

Fig. 2 Measurement-identification error relation for truss (GPE).

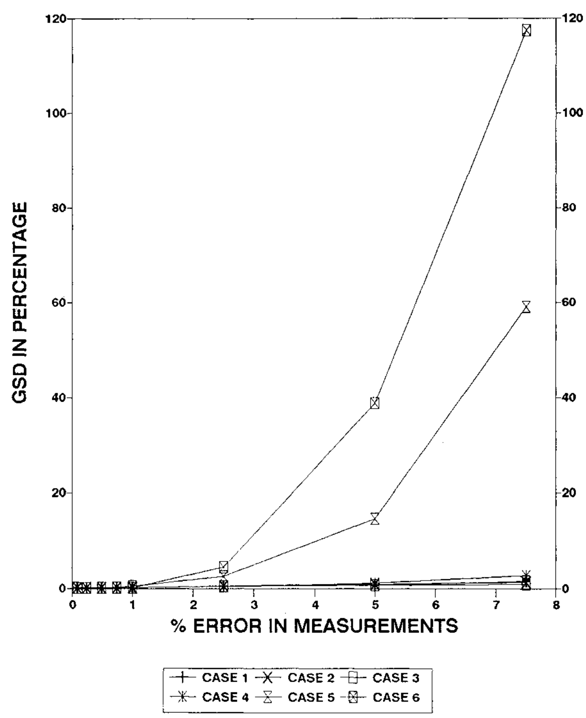

Fig. 3 Measurement-identification error relation for truss (GSD).

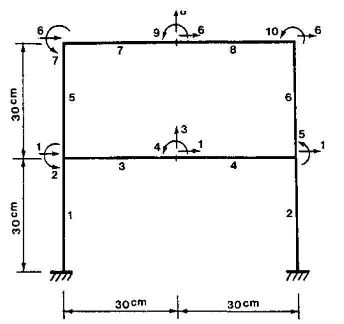

Fig. 4 Two-story frame.

ple) is reported for the frame example in Table 10. MPEA is the maximum percentage error for all identified parameters for a 1% measurement error in all available DOF. The first two columns of Table 10 show all FDOF and DDOF available for measurement. The eliminated DOF are symbolized with a dash and are no longer available for measurement. In addition, when a DOF is eliminated from the set, if the sensitivity matrix become singular or ill-conditioned, it is symbolized with an S.

All FDOF and DDOF are initially assumed available for measurement, and all parameters (i.e., equivalent moment of inertias) are assumed to be unknown. The first row of the data in Table 10 shows MPEA when none of FDOF and DDOF are eliminated from that case (referred to as sensitivity analysis). The subsequent rows show MPEA when one of the DOF is eliminated from the set of measured FDOF or DDOF (referred to as drop analysis). The first two columns of the table show the DOF eliminated in each case to examine the effect of the measurement errors on the parameter estimates. The next seven columns show MPEA for cases 1-7. Since it is desirable to limit the maximum error in the parameters, MPEA is ranked to drop the least error-sensitive DOF. Table 10 is the result of seven computer runs representing the error sensitivities when different DOF are eliminated from each case. Case 1 shows that, if all DOF are measured with a 1% input error, MPEA is −42.0%. As FDOF and DDOF are eliminated one by one, MPEA varies from −1.8 to −58.3%. The degrees of freedom that by their elimination have the smallest impact on the identification error are DDOF 1, 3, 6, and 8 with MPEAs <2.5%. It should be noted that the eliminated DOF are translational displacements.

Case 2 shows the results of a subsequent error sensitivity analysis after DDOF 1, 3, 6, and 8 are eliminated. For the remaining 10 forces and 6 displacements MPEA is −2.7%, which is an immense reduction compared to case 1. Based on the drop analysis of case 2, the best DOF to eliminate is FDOF 6 with an MPEA of −2.0%.

Case 3 shows that measuring the remaining nine forces and six displacements will further reduce MPEA to −2.0%. Next, dropping FDOF 3 will cause an MPEA of −1.8%.

Case 4 lists the results of the error sensitivity analysis with FDOF 3 dropped along with all the previously dropped DOF. MPEA for the remaining measured DOF is further reduced to −1.8%. The drop analysis reveals that it is best to drop FDOF 2, 4, 5, 7, 9, and 10 (all bending moments) with an MPEA of −1.8%. The worst DOF to eliminate is FDOF 8.

Case 5 shows the error sensitivity analysis results with the remaining measured FDOF of 1 and 8, and DDOF of 2, 4, 5, 7, 9, and 10. A 1% measurement error will cause an MPEA of −1.8% in the parameter estimates. The subsequent drop analysis show that dropping FDOF 1 or 8 is not feasible. It makes the sensitivity matrix singular due to the reduction of the number of independent measurements (NIM) to 6, which is less than NUP of 8. Dropping the remaining DOF shows that, if DDOF 7 or 10 is eliminated, it will cause a −2.3% error. In a run not reported here both DDOF 7 and 10 were eliminated and caused an ill-conditioned sensitivity matrix. Therefore, it was decided to eliminate only DDOF 7.

Case 6 shows that, when DDOF 7 is dropped, MPEA is −2.3%. The subsequent drop analysis shows that FDOF 1 or 8 and DDOF 9 or 10 are not to be eliminated. However, it is possible to drop DDOF 4 which will cause a −5.0% error.

Case 7 shows that MPEA is −5.0%, and if any of the remaining DOF are eliminated, NIM will become less than NUP of 8, not allowing for a unique parameter estimation. Case 7 contains the smallest set of FDOF and DDOF that can be used for a unique parameter estimation. The NIM for cases 1-7 reduce from 55 to 8 and are listed in Table 11.

The first row of Table 10 shows that the identification error initially was very high (−42%). Then by eliminating the least error-sensitive DOF, the maximum error reduced to −1.8% in cases 4 and 5 and increased to −5.0% in case 7. This simulation shows that measuring more DOF does not necessarily result in a more accurate parameter estimates.

Cases 2-7 give acceptable sets of DOF for measurement that allow identification with reasonably low errors in the parameter estimates. A small set of measurements is used in cases 5-7

Table 10 Results of frame error sensitivity analysis

Eliminated		MPE for a 1% measurement error						
FDOF	DDOF	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7
0	0	−42.0	−2.7	−2.0	−1.8	−1.8	−2.3	−5.0
1	0	−48.7	−4.3	3.4	6.7	S	S	S
2	0	−58.3	−2.7	−2.0	−1.8	—	—	—
3	0	−33.1	−3.1	−1.8	—	—	—	—
4	0	−47.4	−2.7	−2.0	−1.8	—	—	—
5	0	−58.3	−2.7	−2.0	−1.8	—	—	—
6	0	−47.5	−2.0	—	—	—	—	—
7	0	−51.2	−2.7	−2.0	−1.8	—	—	—
8	0	−29.1	2.9	−2.9	33.9	S	S	S
9	0	−46.9	−2.7	−2.0	−1.8	—	—	—
10	0	−51.2	−2.7	−2.0	−1.8	—	—	—
0	1	−2.4	—	—	—	—	—	—
0	2	−37.3	−6.4	−4.0	−5.2	−5.2	−7.7	S
0	3	−1.8	—	—	—	—	—	—
0	4	−44.3	−2.1	−2.5	−3.3	−3.3	−5.0	—
0	5	−37.3	−6.4	−4.0	−5.3	−5.2	−6.5	S
0	6	−2.0	—	—	—	—	—	—
0	7	−34.7	−3.1	−2.1	−2.3	−2.3	—	—
0	8	−2.2	—	—	—	—	—	—
0	9	−44.3	−2.6	−2.0	−2.6	−2.6	S	S
0	10	−34.7	−3.1	−2.1	−2.3	−2.3	S	S

Table 11 Frame identification error percentages for all cases (measurement error = 1%)

Case	Measured FDOF	Measured DDOF	NIM	GPE	GSD
1	1, 2, 3, 4, 5, 6, 7, 8, 9, 10	1, 2, 3, 4, 5, 6, 7, 8, 9, 10	55	62.913	0.031
2	1, 2, 3, 4, 5, 6, 7, 8, 9, 10	2, 4, 5, 7, 9, 10	45	0.210	0.104
3	1, 2, 3, 4, 5, 7, 8, 9, 10	2, 4, 5, 7, 9, 10	39	0.142	0.077
4	1, 2, 4, 5, 7, 8, 9, 10	2, 4, 5, 7, 9, 10	33	0.130	0.068
5	1, 8	2, 4, 5, 7, 9, 10	12	0.114	0.079
6	1, 8	2, 4, 5, 9, 10	10	0.121	0.090
7	1, 8	2, 5, 9, 10	8	0.169	0.168

and yet results in low errors. In order to select the best case among these cases and also determine the level of acceptable measurement error, a Monte Carlo experiment is performed for each case to study the input/output error behavior.

Table 11 shows the results of the Monte Carlo analysis given a ± 1% uniform proportional measurement error for each of the seven cases presented. One hundred observations are performed for each case. All observations for these seven experiments converged in less than five iterations. The percentage errors in all cases are low and reasonable, except in case 1. This will allow comparisons between the parameter estimates and their initial values for damage assessment.

Similar to the truss example, Monte Carlo experiments are performed for these cases at different input error levels. The input/output error relations are used to select one of the cases presented in Table 11 and to determine the acceptable input error for potential NDT. For brevity, these results are not included.

Since a small set of measurements is desirable, case 7 is recommended for potential NDT. It uses only eight independent measurements to identify all eight unknown moment of inertias. Using several Monte Carlo experiments at various input error levels, the measurement error was selected at 2.5%, which resulted in GPE and GSD of 0.392 and 0.451%, respectively. All 100 observations converged in less than three iterations for case 7.

Case 7 uses two force measurements (no bending moments) and 4 rotation measurements (no translations). Rotations can be measured using electrolytic tilt transducers with sensitivities as great as one part in million. If some of the DOF are not available for measurement, they can be eliminated from the set of DOF prior to the start of the drop analysis.

Again, the two-story frame example illustrates that the error sensitivity analysis is an essential means for pretest selection of a subset of DOF for measurements. This will ensure a successful parameter estimation. Using the error sensitivity algorithm presented in this paper, significantly better subsets of DOF (cases 5–7) were chosen compared to case 1 for parameter estimation.

The frame example presented is different from the truss example in terms of element types, boundary conditions, dimensionality, loading conditions, measurements, and identified parameters. Both the truss and the frame examples resulted in similar trends in the error sensitivity analysis, which led to a successful selection of a subset of DOF for NDT.

Although the error sensitivity analysis and the subsequent Monte Carlo analysis are computing intensive, it is better than arbitrarily trying a large number of subsets of FDOF and DDOF in search of a case in which the input/output error relation is probably well behaved. Additionally, after the pretest selection of FDOF and DDOF for NDT, only a single parameter identification run is required, either for damage assessment or for health monitoring of the structure. A single identification run using test data is substantially less computing intensive than the drop analysis or the Monte Carlo analysis performed in this paper.

Conclusions

In the presence of measurement errors the structural element stiffnesses were identified successfully from complete or incomplete static test data. By applying forces to a subset of DOF and measuring displacements at another subset of DOF, reliable estimates of the structural parameters were obtained. The DOF selected for measurements play an essential role in successful identification of the unknown parameters.

An error sensitivity analysis was performed to select noisy subsets of DOF for measuring applied forces and displacements that will cause small errors in the identified parameters. Using simulated measurements at the selected DOF, the relationship between the measurement errors and the errors in the identified parameters was established. Using this relationship, given the error tolerance in the identified parameters that will allow damage assessment, an acceptable measurement error level was determined.

As a result of applying this algorithm, it is concluded that key factors for successful parameter identification are the number of the measurements, the locations of the measurements, and the precision of the measurements. A pretest error sensitivity analysis will result in the selection of a set of error-tolerant DOF for measurement that can be used for parameter identification and damage assessment.

Future work could include testing of small-scale structures to validate this algorithm. After laboratory testing is completed and the method is expanded to include various structural finite elements, it can then be used for damage assessment and health monitoring of full-scale structures. Additionally, a study of practical turnaround times for large structures is of paramount importance.

Acknowledgment

The authors gratefully acknowledge the suggestions of Professor Linfield C. Brown in formulating the algorithm for the error sensitivity analysis.

References

[1]Baruch, M., "Optimal Correction of Mass and Stiffness Matrices Using Measured Modes," *AIAA Journal*, Vol. 20, No. 11, 1982, pp. 1623–1626.

[2]Berman, A., "System Identification in Structural Dynamic Models—Theoretical and Practical Bounds," AIAA Structures, Structural Dynamics and Materials Conference, AIAA Paper 84-0929, Palm Springs, CA, 1984, pp. 123–129.

[3]Kabe, A. M., "Stiffness Matrix Adjustment Using Mode Data," *AIAA Journal*, Vol. 23, No. 9, Sept. 1985, pp. 1431–1436.

[4]Adelman, H. M., and Haftka, R. T., "Sensitivity Analysis of Discrete Structural Systems," *AIAA Journal*, Vol. 24, No. 5, May 1986.

[5]Beck, J. L., "Determining Models of Structures from Earthquake Records," California Inst. of Technology, EERL Rept. 78-01, Pasadena, CA, June 1987.

[6]Chen, J. C., and Garba, J. A., "On Orbit Damage Assessment for Large Space Structures," 28th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, Monterey, CA, 1987.

[7]Stubbs, N., Broome, T. H., and Osegueda, R., "Nondestructive Construction Error Detection in Large Space Structures," *AIAA Journal*, Vol. 28, No. 1, Jan. 1990, pp. 146–152.

[8]Hajela, P., and Soeiro, F. J., "Recent Developments in Damage Detection Based on System Identification Methods," *Structural Optimization*, Vol. 2, 1990, pp. 1–10.

[9]Sanayei, M., and Nelson, R. B., "Identification of Structural Element Stiffnesses from Incomplete Static Test Data," Aerospace

Technology Conference and Exposition, Society of Automotive Engineers Rept. 861793, Long Beach, CA, Oct. 1986.

[10]Sanayei, M., and Onipede, O., "Damage Assessment of Structures Using Static Test Data," *AIAA Journal,* Vol. 29, No. 7, July 1991, pp. 1174–1179.

[11]Robson, B. N., Frangopol, D. M., and Goble, G. G., "Optimization Models for Predicting Structural Response in Highway Bridges," *Electronic Computation, Proceedings of the Tenth Conference,* American Society of Civil Engineers, Indianapolis, IN, April 1991.

[12]Hjelmstad, K. D., Wood, S. L., and Clark, S. J., "Parameter Estimation in Complex Linear Structures," Univ. of Illinois at Urbana-Champaign, UILU-ENG-90-2015, Urbana, IL, Dec. 1990.

[13]Skelton, R. E., and Delorenzo, M. L., "Selection of Noisy Actuators and Sensors in Linear Stochastic Systems," *Journal of Large Scale Systems, Theory and Applications,* Vol. 4, 1983, pp. 109–136.

[14]Haftka, R. T., and Adelman, H. M., "Selection of Actuator Locations for Static Shape Control of Large Space Structures by Heuristic Integer Programming," *Computers and Structures,* Vol. 20, Nos. 1-3, 1985, pp. 575–582.

[15]Kammer, D. C., "Sensor Placement for On-Orbit Modal Identification and Correlation of Large Space Structures," *Journal of Guidance, Control, and Dynamics,* Vol. 14, No. 2, 1991, pp. 251–259.